Permafrost melt and its effects on planetary energy balance

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University of Minnesota

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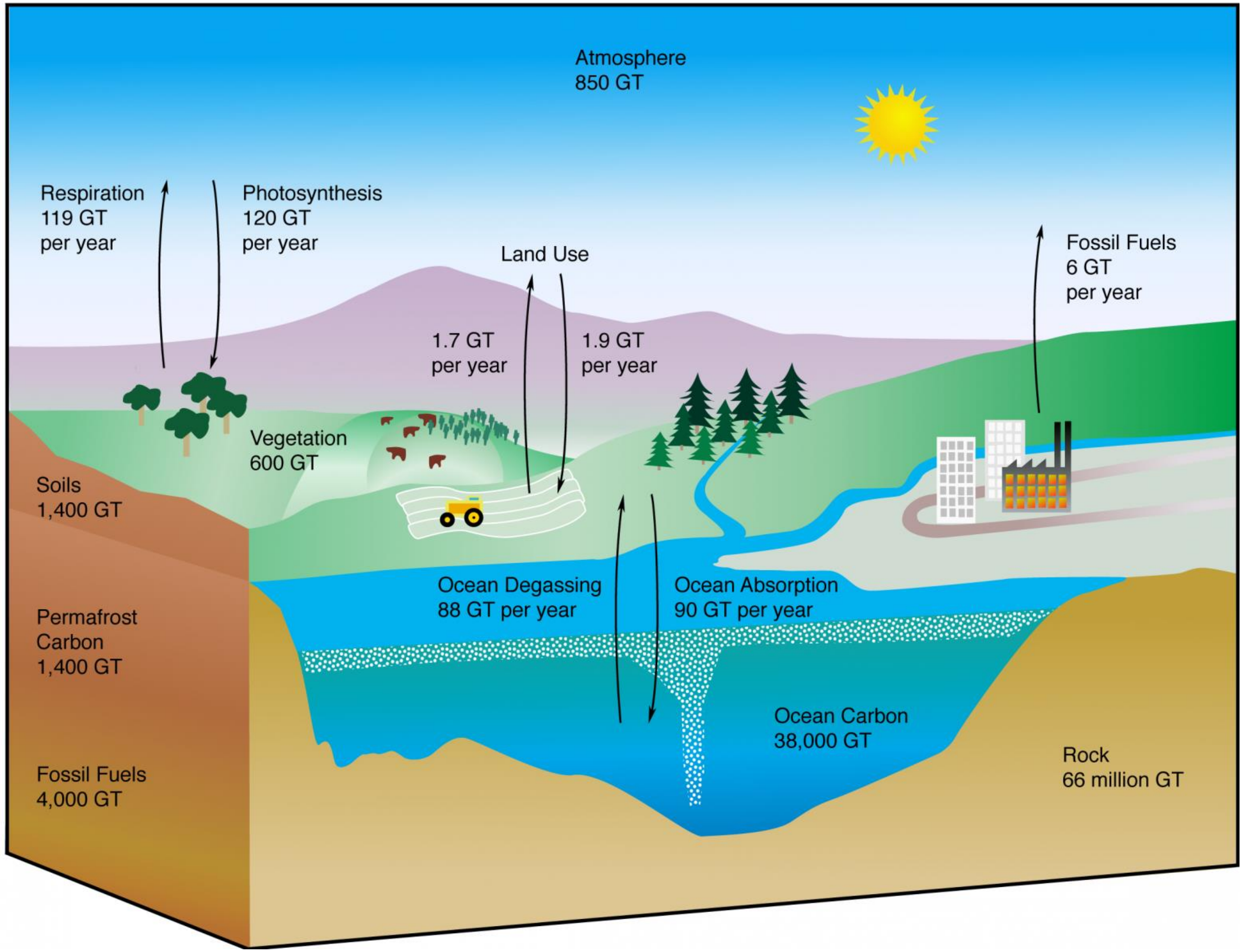
Motivation: the role of permafrost in global carbon cycle

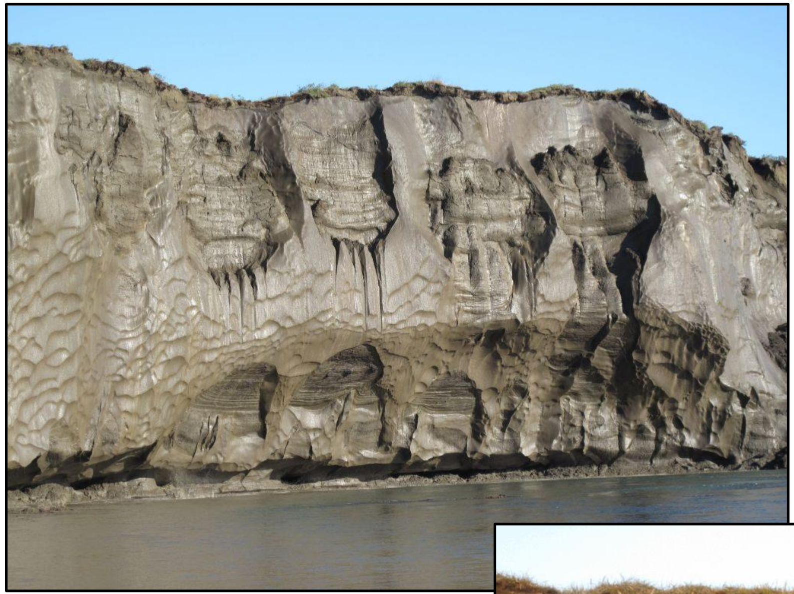


(NSIDC)

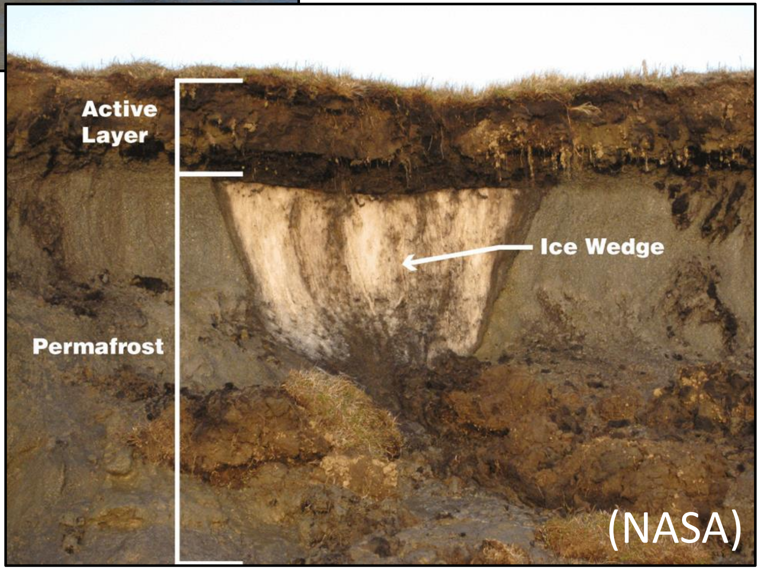
Motivation: the role of permafrost in global carbon cycle

Major characteristics of permafrost:

- Frozen soil/ice composite
- $\leq 0°C$ for at least two years
- Active layer:
  - top portion melting/refreezing
- Maximum depth:
  - 500 m (modern)
  - 1,000 m (paleo)



(Arctic Today)



Motivation: the role of permafrost in global carbon cycle

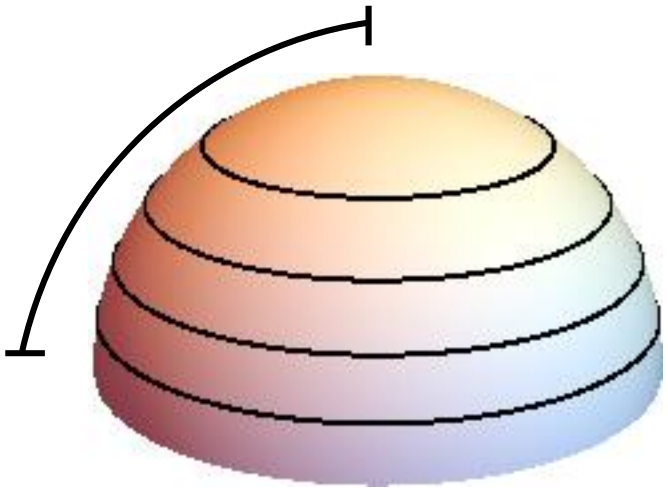
Permafrost Type

- Isolated
- Sporadic
- Discontinuous
- Continuous

(Schuur et al. 08)

Outline

- Budyko's energy balance model for surface temperature
- Linear approximation to changes in permafrost
- Explicit model for heat conduction through soil
- Future work



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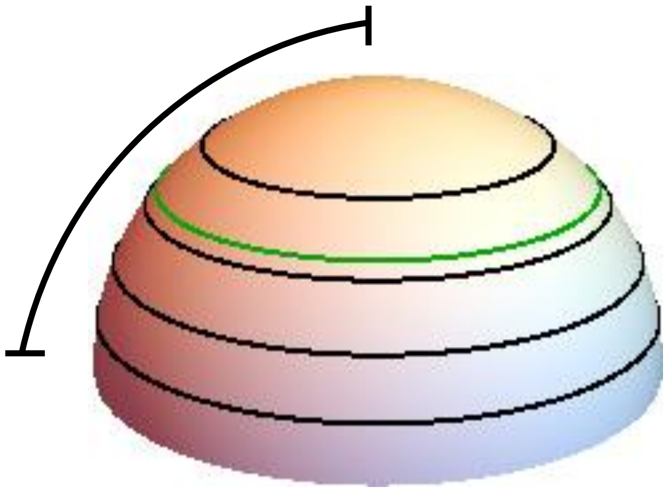

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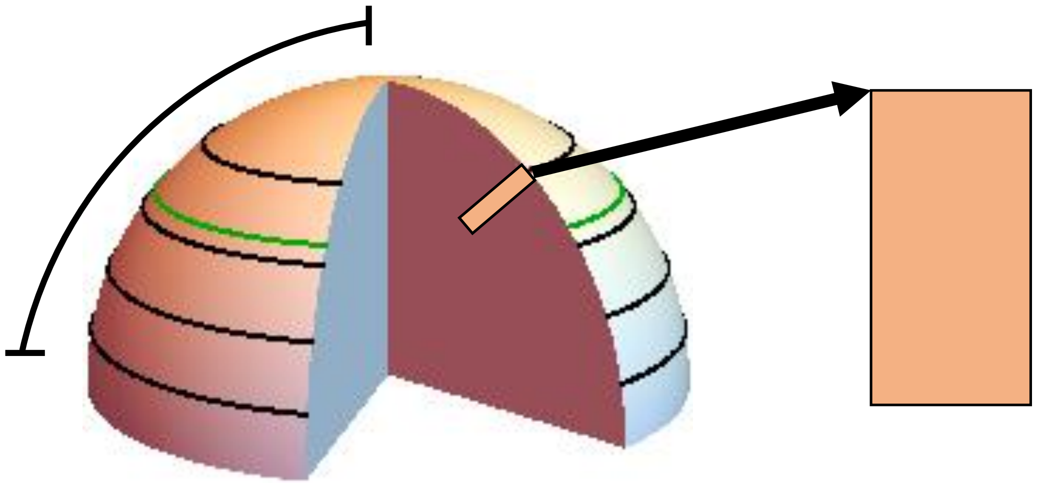

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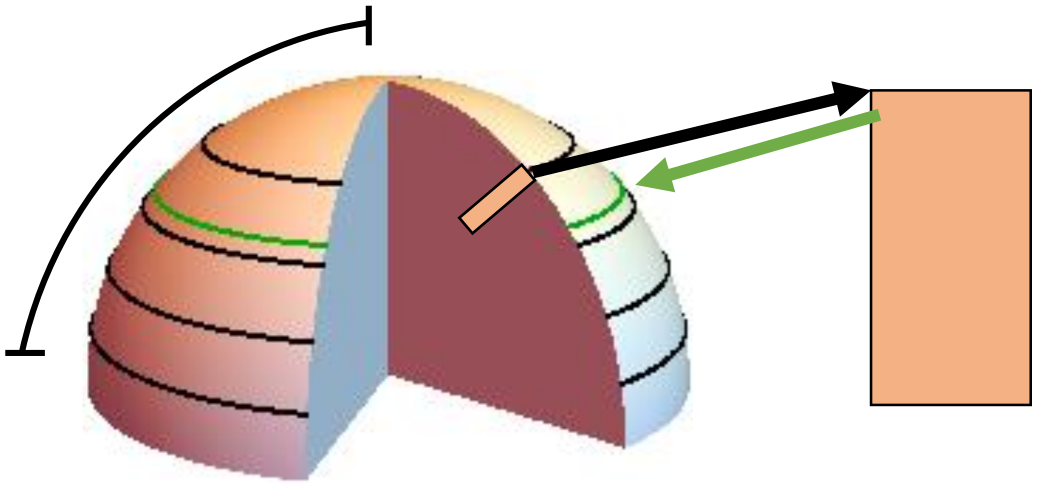
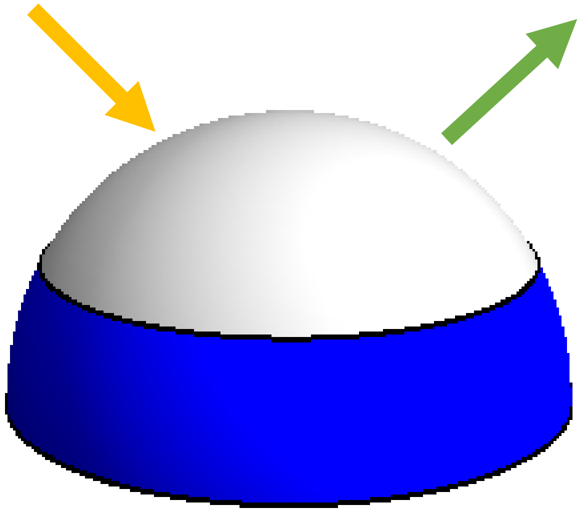


Surface Temperature: Budyko's energy balance model

Model surface energy balance using temperature:

$$\frac{\partial T}{\partial t} = \text{Energy in} - \text{Energy out}$$



(Budyko 69)

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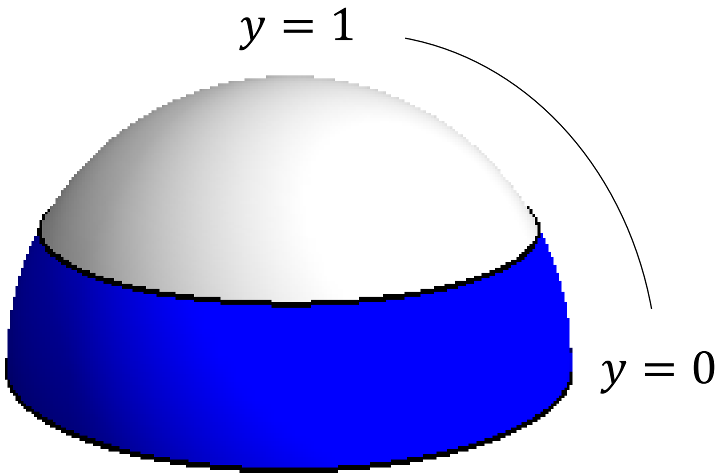
$$R\,\frac{\partial T}{\partial t}(y,t) = \underbrace{\big(1-\alpha(y,\eta)\big)}_{\text{albedo}}\underbrace{Qs(y)}_{\text{incoming solar radiation}} \;-\; \underbrace{(A+BT)}_{\text{outgoing longwave radiation}} \;-\; \underbrace{C(T-\bar{T})}_{\text{heat transport}}$$

albedo, incoming solar radiation, outgoing longwave radiation, heat transport

$y = \sin(\text{latitude})$



(Budyko 69)

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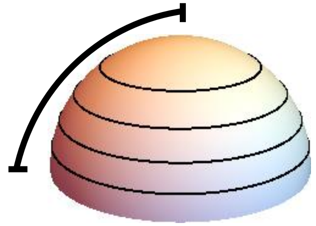

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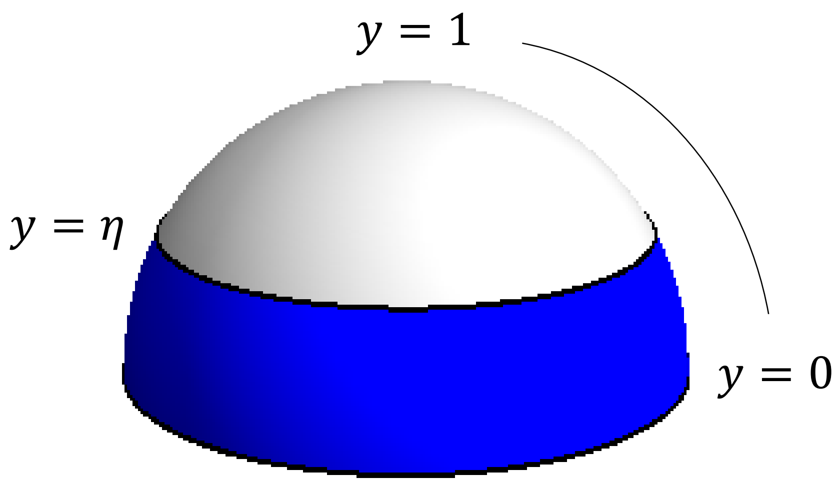
$$\alpha(y,\eta) = \begin{cases} \alpha_1\,, & y>\eta \quad \text{[ice]} \\ \alpha_2\,, & y<\eta \quad \text{[not ice]} \end{cases}$$

$$s(y) \approx 1 + s_2\,(3y^2+1)$$

$y = \sin(\text{latitude})$

$\eta = $ ice line



(Budyko 69, North 75)

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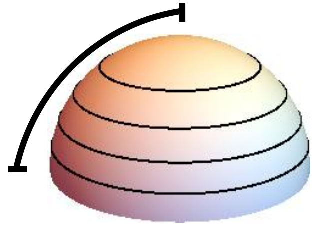

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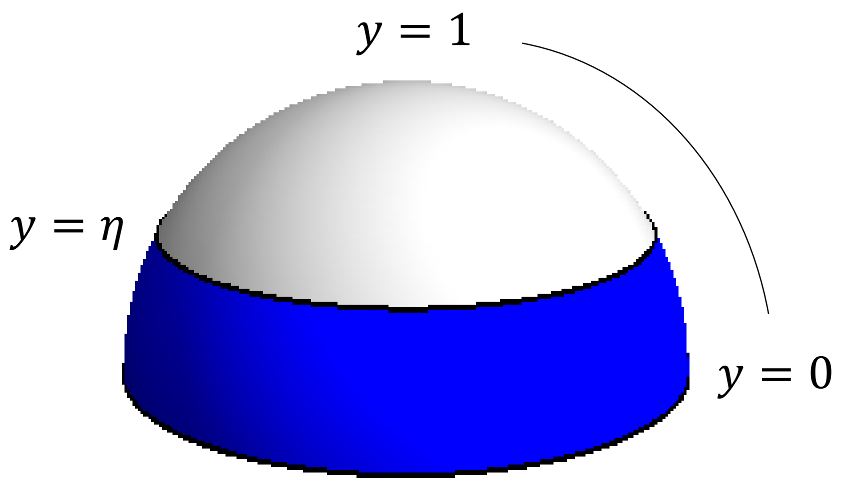
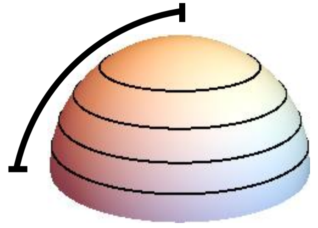
$$\bar{T} = \int_0^1 s(y)dy$$



(Budyko 69, North 75)

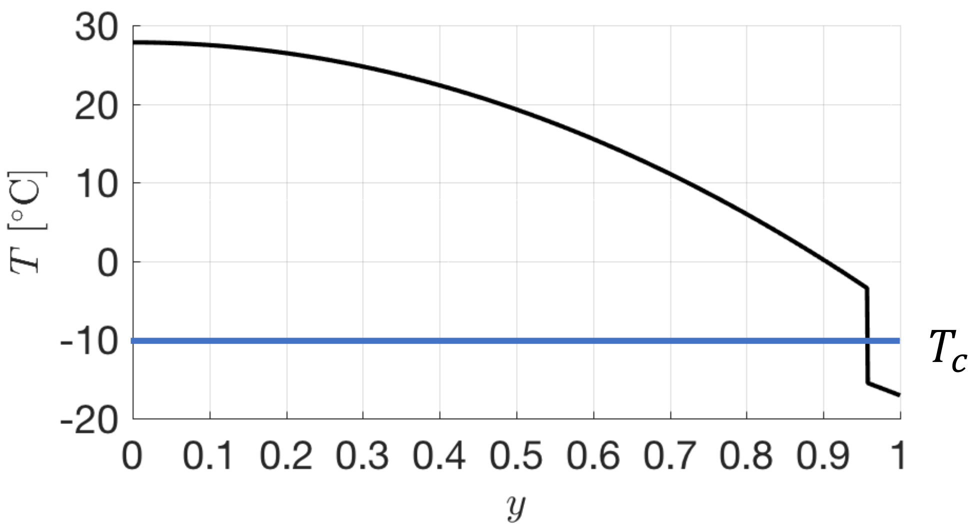
Approximating emissions from permafrost



Steady-state solutions to Budyko's model are given by:

$$T^*(y) = \frac{1}{B+C}\Big(Qs(y)\big(1-\alpha(y)\big) - A + C\bar{T}\Big)$$

$$= \begin{cases} \frac{1}{B+C}(Qs(y)(1-\alpha_1) - A + C\bar{T}), & T > T_c \\ \frac{1}{B+C}(Qs(y)(1-\alpha_2) - A + C\bar{T}), & T < T_c \end{cases}$$



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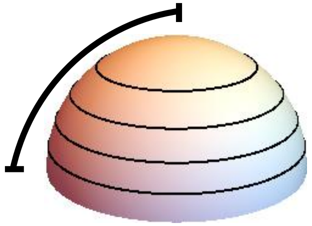

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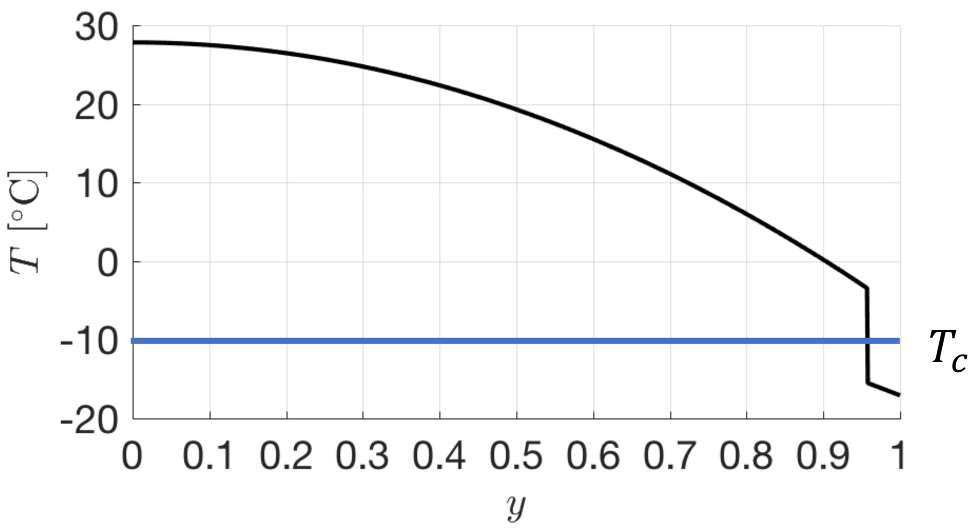

Motivating question:

As permafrost melts, how much carbon dioxide and methane is released?

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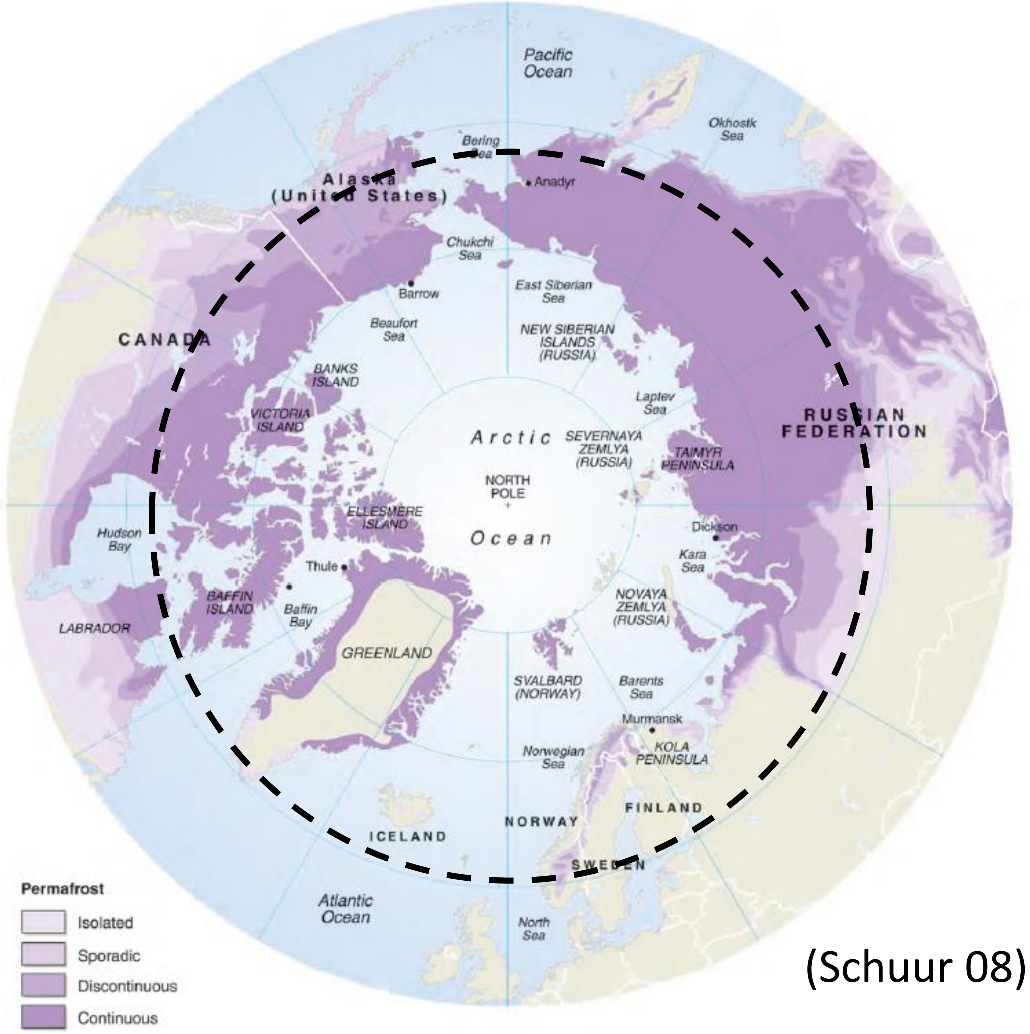
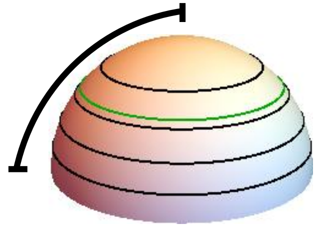


(Schuur 08)

Approximating emissions from permafrost

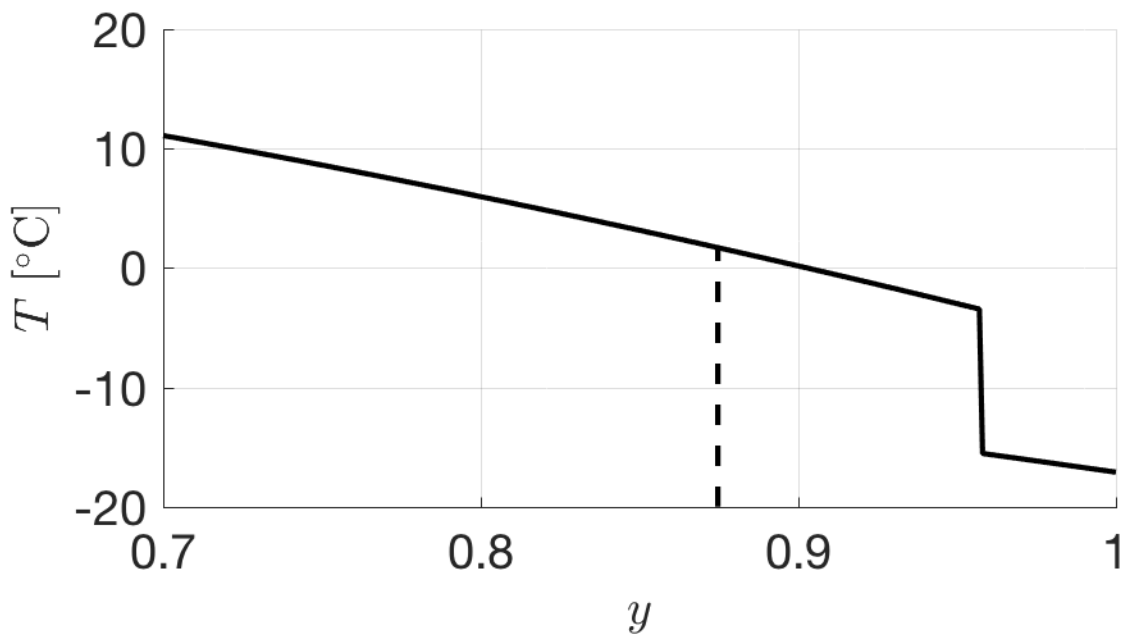


Rate of change of permafrost line with global mean temperature:

$$T^*(p) = \frac{1}{B+C}\left(Qs(p)(1-\alpha_1) - A + C\bar{T}\right)$$

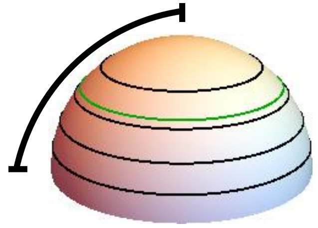
$$\rightarrow \quad \frac{dp}{d\bar{T}} = -\frac{C}{Qs'(p(\bar{T}))(1-\alpha_1)}$$

$$\rightarrow \quad \Delta p \approx p'(\bar{T})\Delta\bar{T}$$



(Zebrowski and Nguyen, *in prep*)

Approximating emissions from permafrost



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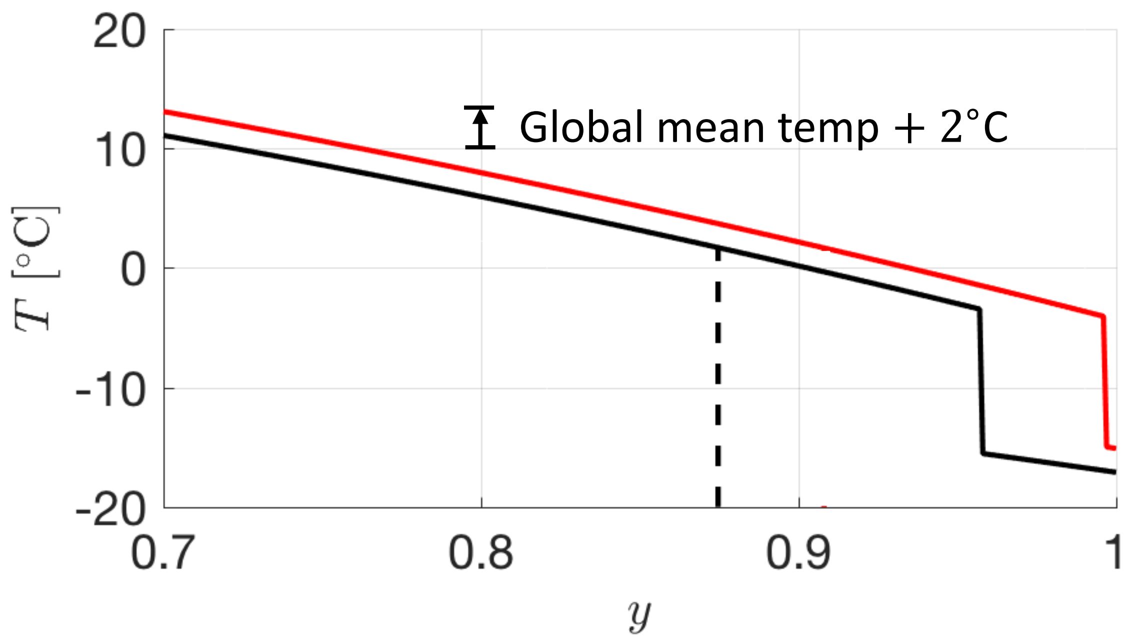

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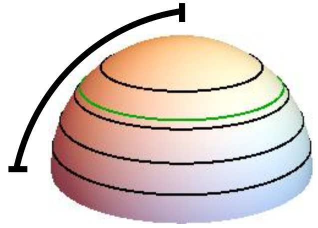

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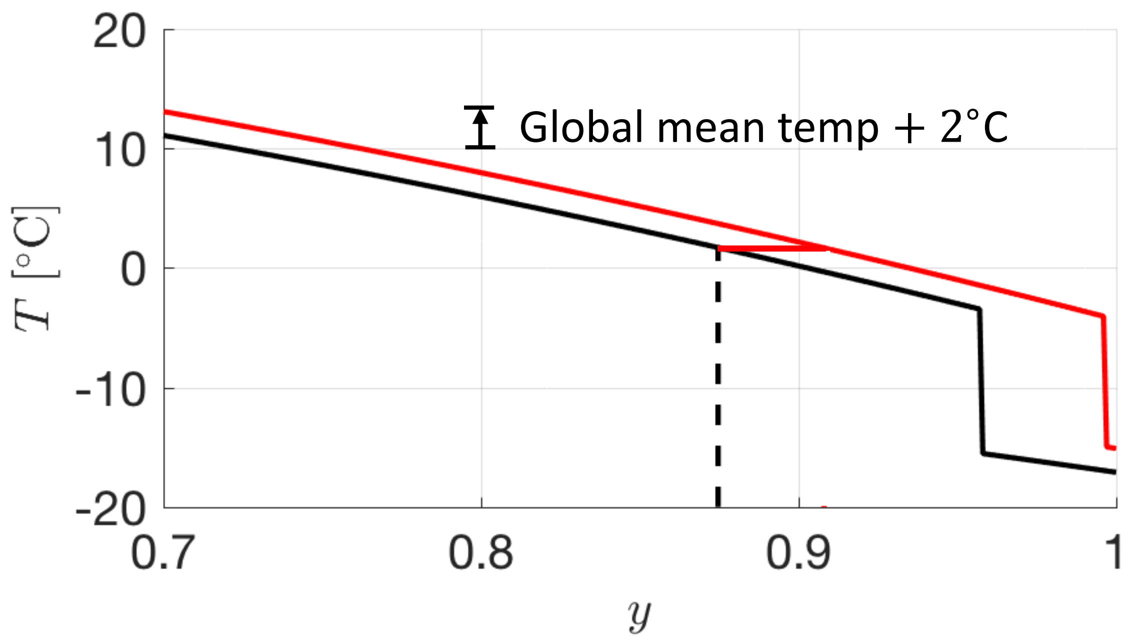

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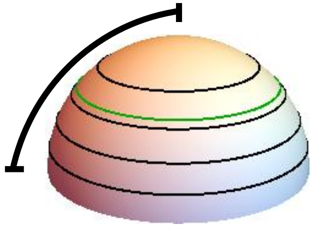

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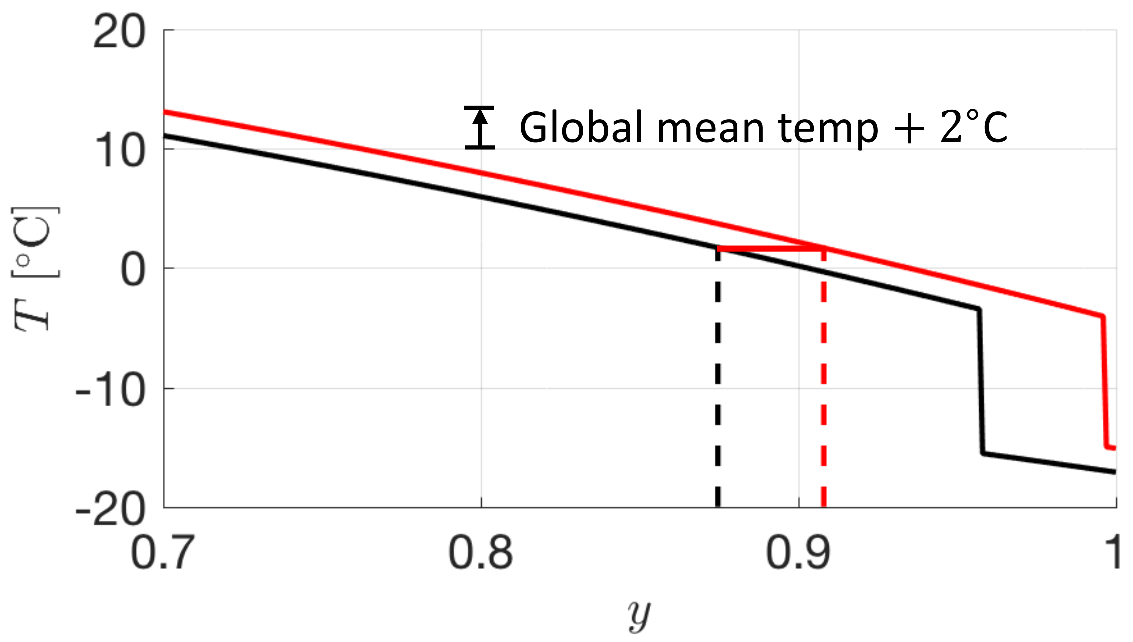

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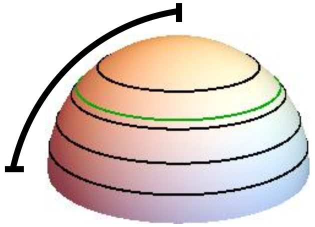


Zebrowski and Nguyen predict 309.7 Gt C released using Budyko's model and geometric arguments

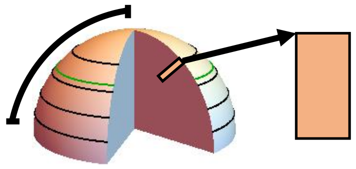
Comparison to:

Table 2. Projections of cumulative emissions from thawing permafrost, with CO_2 equivalents in parentheses[a].

Study	2100	Permafrost carbon emissions (Gt C)2200	2300	Flux uncertainty (%)	Temperature increase (K)2100	Initial carbon stock (Gt C)	Permafrost area loss (%)2100	Scenario
Zhuang *et al* (2006)[b]	37 (46)	na[c]	na	3%	na	na		A2
Dutta *et al* (2006)	40 (50)	na	na	na	na	460		5 °C Siberia
Burke *et al* (2013)	50 (62)[e]	na	99 (124)[e]	41%	na	850	76 ± 20	RCP8.5
Schuur *et al* (2013)	158 (198)	na	345 (432)	24%	na	1488	55 ± 5[a]	RCP8.5
MacDougall *et al* (2012)	174 (218)	na	na	61%	0.27 ± 0.16	1026	56 ± 3	RCP8.5
Harden *et al* (2012)	218 (273)[e]	na	436 (546)[e]	85%	na	1060	74	RCP8.5
Raupach and Canadell (2008)[d]	347 (435)	na	na	na	0.7	500		A2

(Schaefer et al 14)

An explicit model for permafrost melt

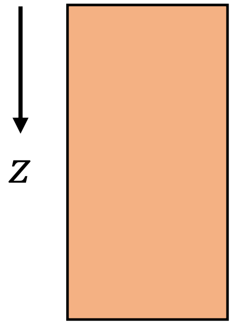


At each latitude, we assume temperature varies by depth via conduction:

$$\frac{\partial T_y}{\partial t} = k \frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$y = \sin(\text{latitude})$

$z =$ soil depth



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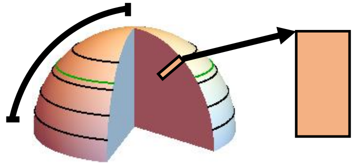

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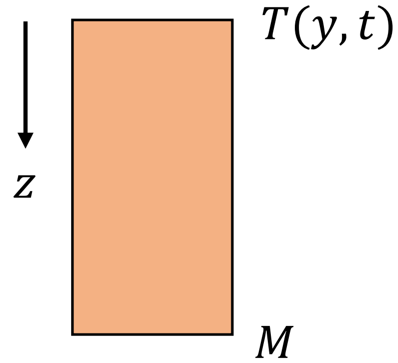
At surface boundary: $T_y(0, t; y) = T(y, t)$

At the lower boundary: $T_y(L, t; y) = M$



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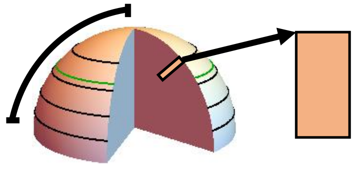

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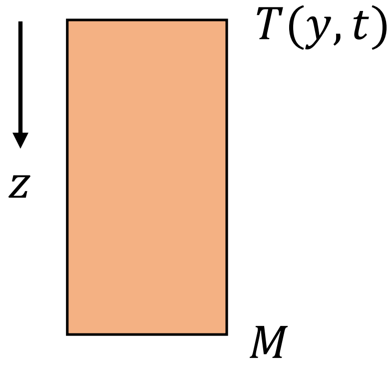
At surface boundary: $T_y(0, t; y) = T(y, t)$

At the lower boundary: $T_y(L, t; y) = M$

Initial condition: $T_y(z, 0; y) = \frac{M - T(y,0)}{L} z + T(y, 0)$



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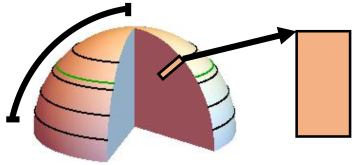
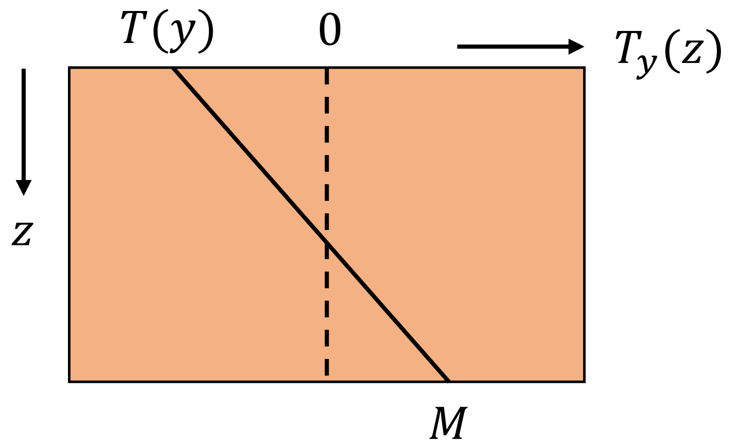


In equilibrium,

$$T_y(z) = \frac{M - T(y)}{l} z + T(y)$$



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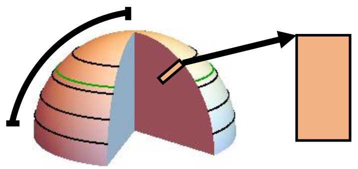

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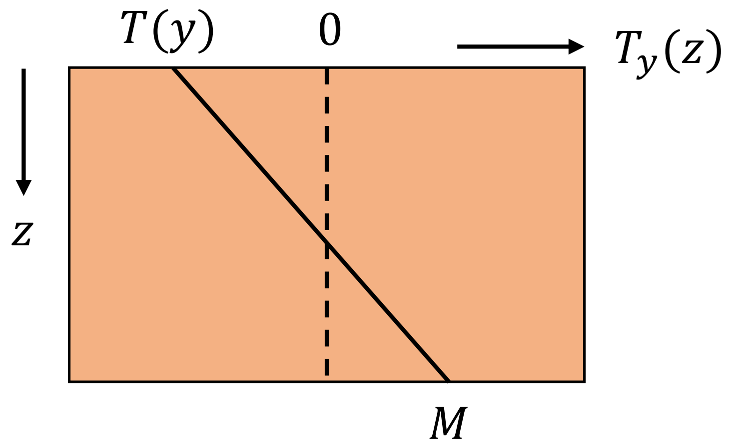

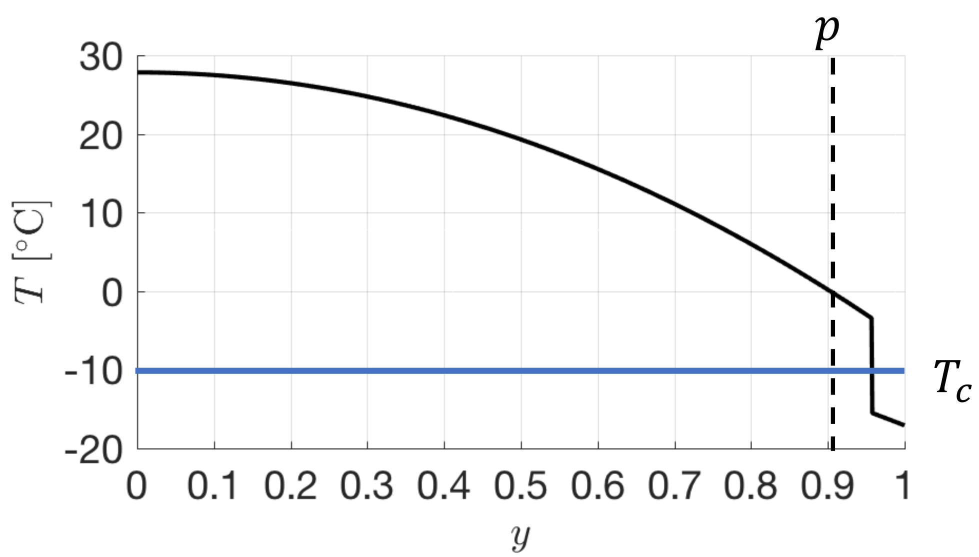

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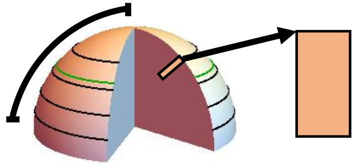

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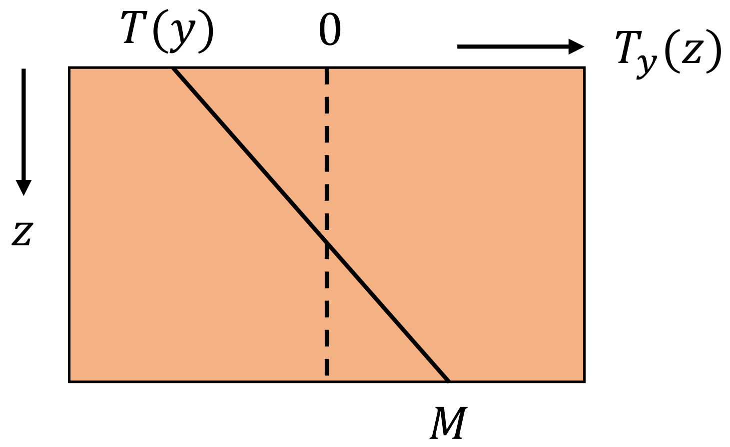
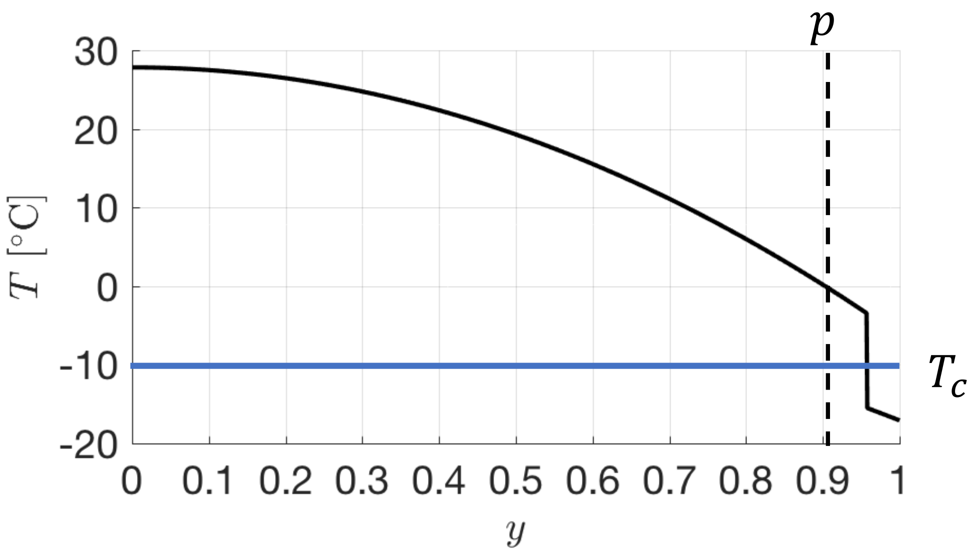
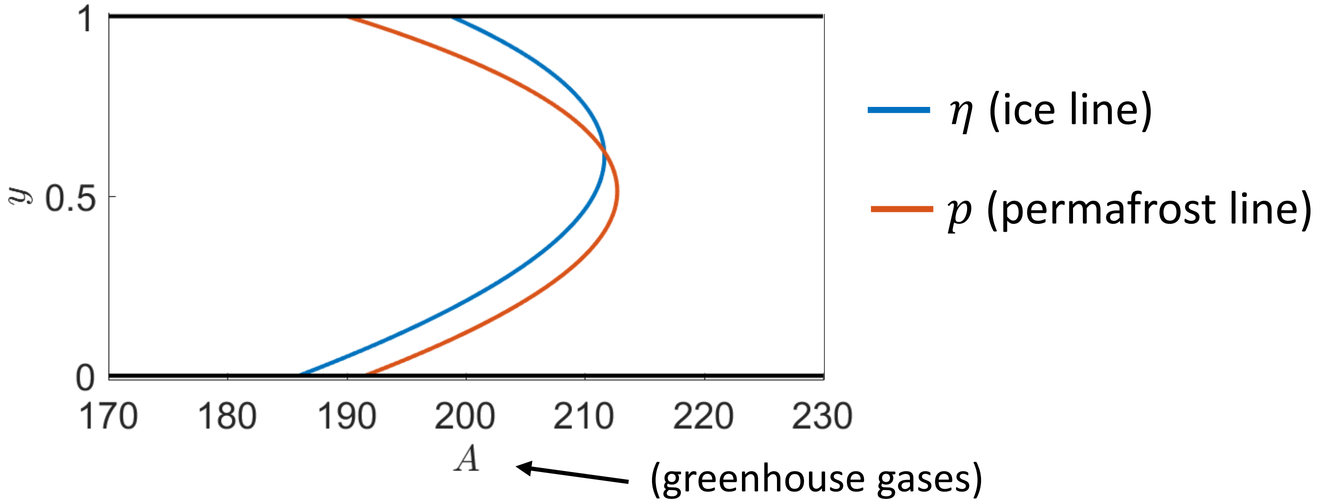
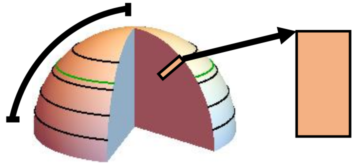






Reality check: how good is the heat equation?



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- Frozen soil/ice composite
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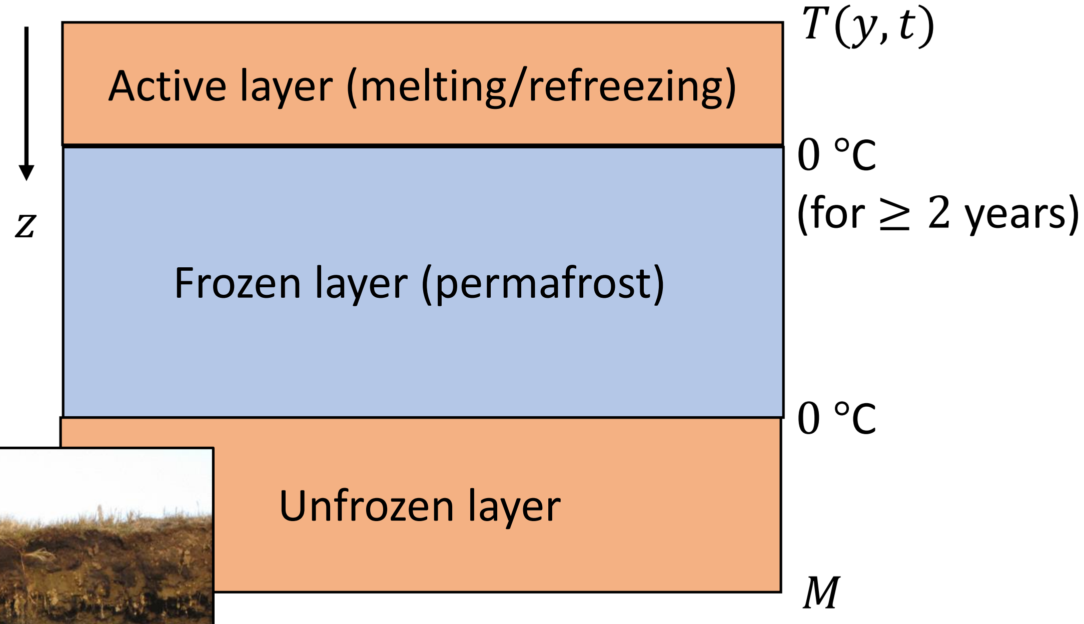
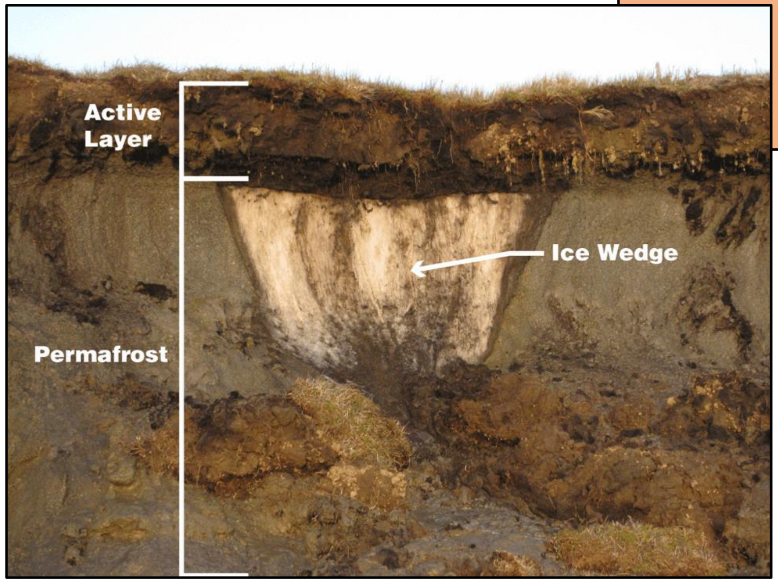




Heat conduction as a model for permafrost

Is it reasonable to model permafrost as heat conduction?

On a decadal timescale, yearly variations may be important:

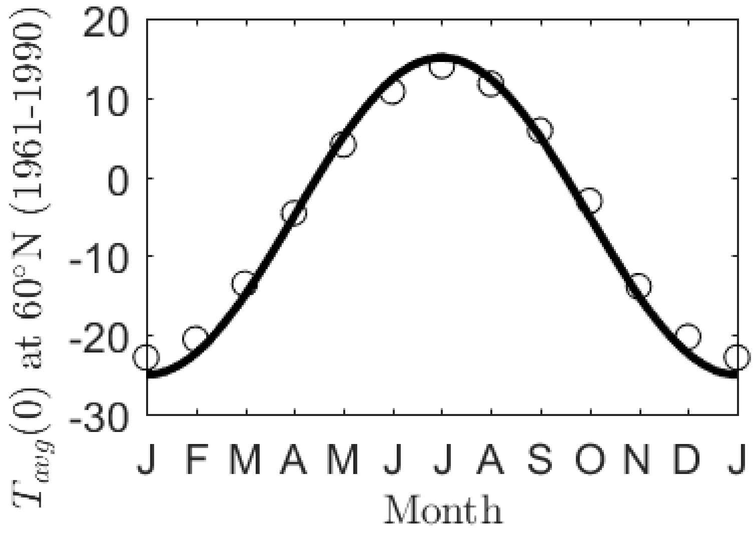
$$\frac{\partial T_y}{\partial t} = k\frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$$T_y(0,t) = T(y,t) \approx (-5 - 20\cos(2\pi t))$$

At 60°N

$$T_y(l,t) = M$$

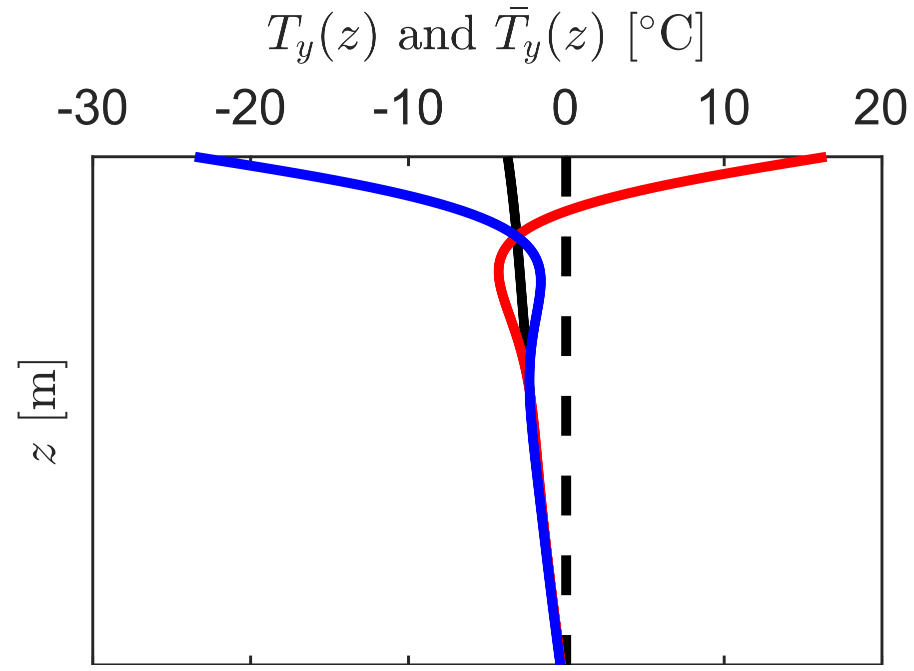
$$T_y(z,0) = \frac{M - T(y,0)}{l}z + T(y,0)$$

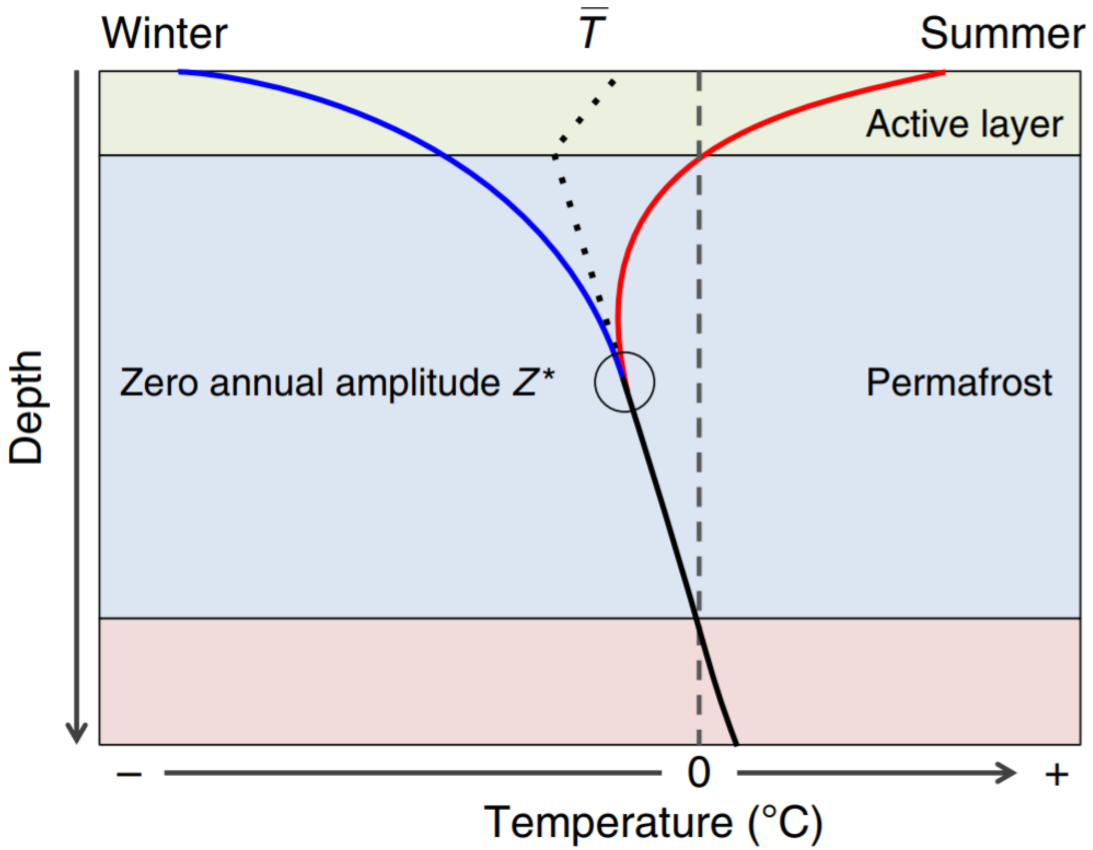


(data: CRU CL v2.0)

Heat conduction as a model for permafrost

The temperature profile has similar characteristics to permafrost:





(Biskaborn et al. 19)

Heat conduction as a model for permafrost

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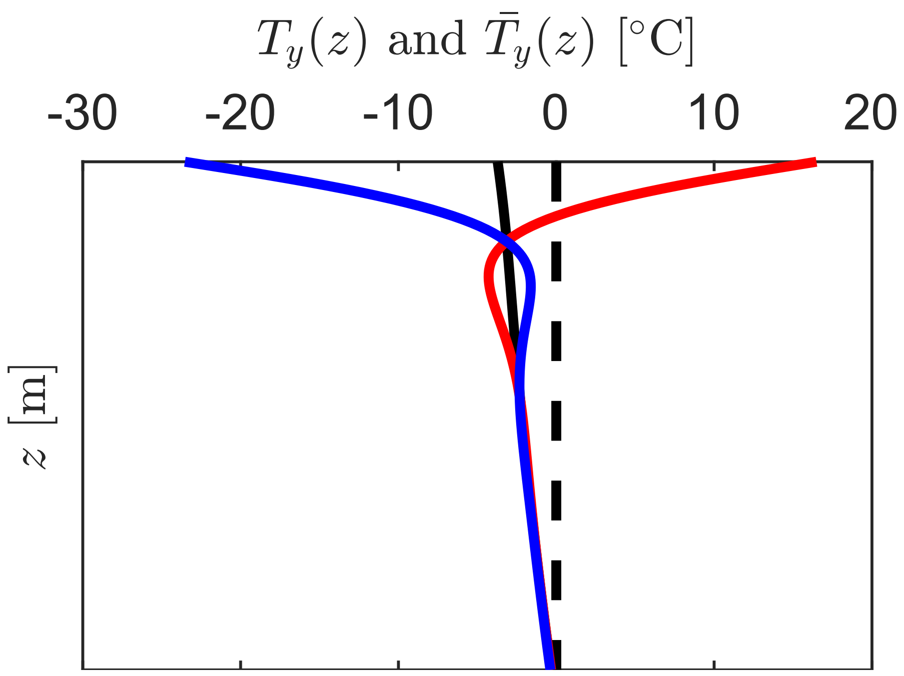
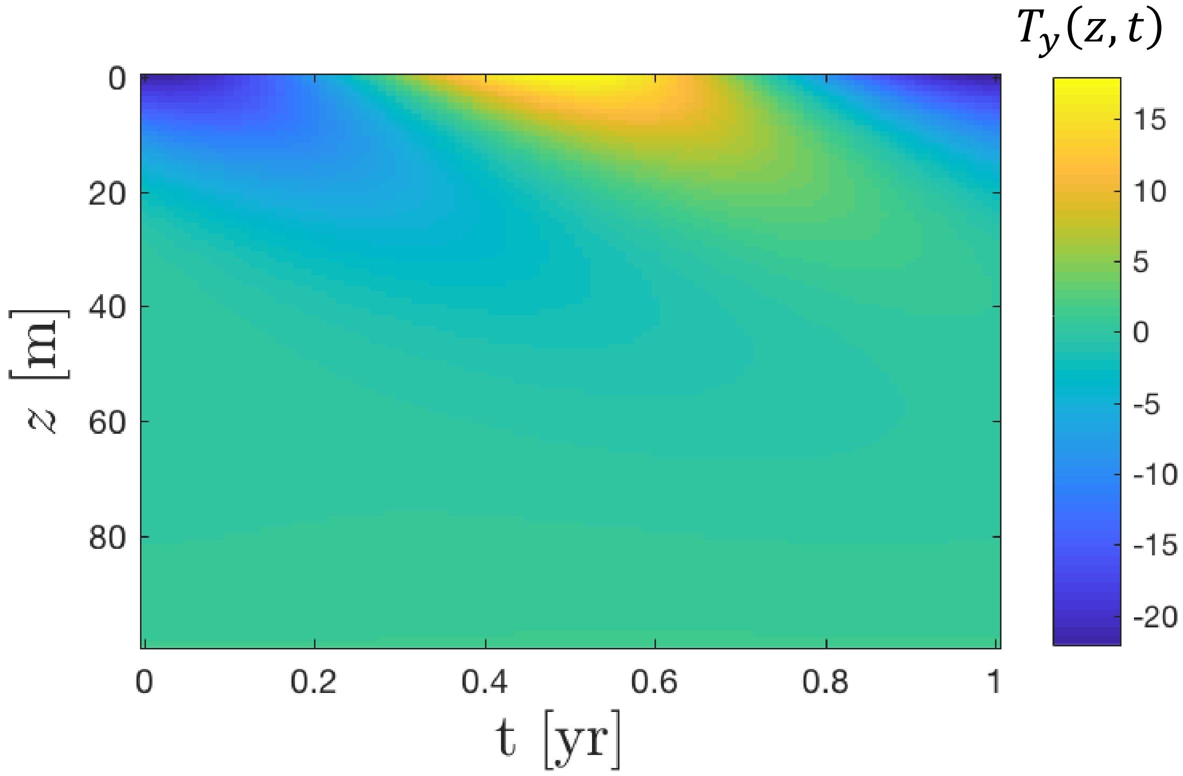




Heat conduction as a model for permafrost

With added forcing, we can simulate the permafrost melting:

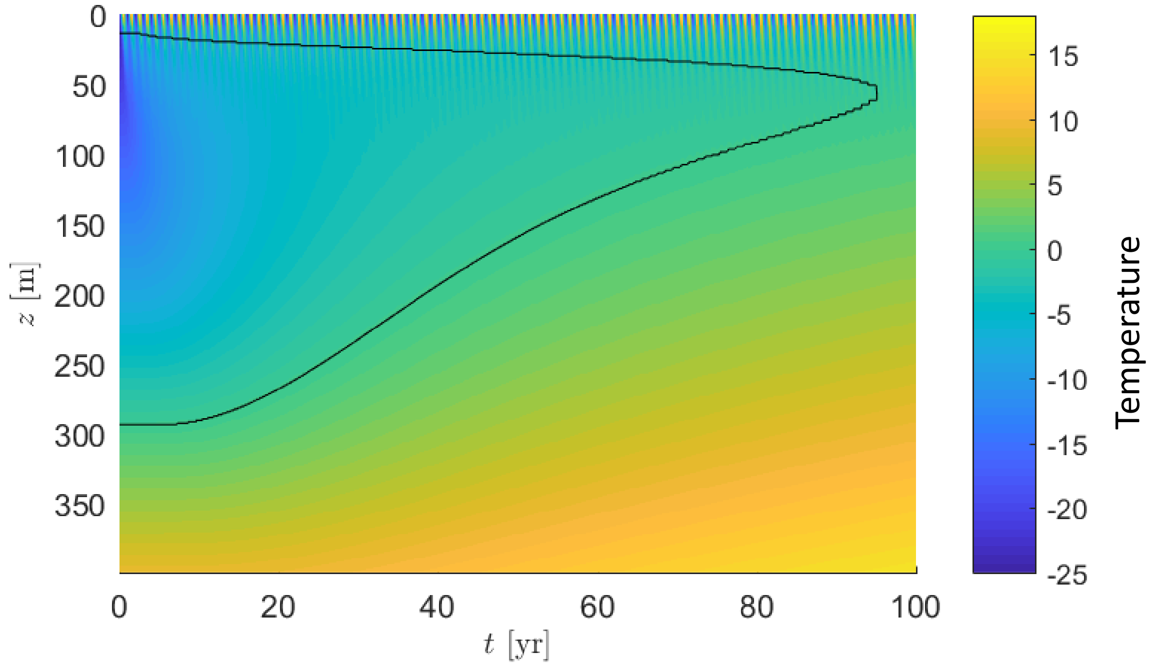
$$\frac{\partial T_y}{\partial t} = k\frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$$T_y(0,t) = T(y,t) + F(t) \approx (-5 - 20\cos(2\pi t)) + \frac{F_{max}t}{t_{max}}$$

$$T_y(L,t) = M$$

$$T_y(z,0) = \frac{M - T(y,0)}{L}z + T(y,0)$$

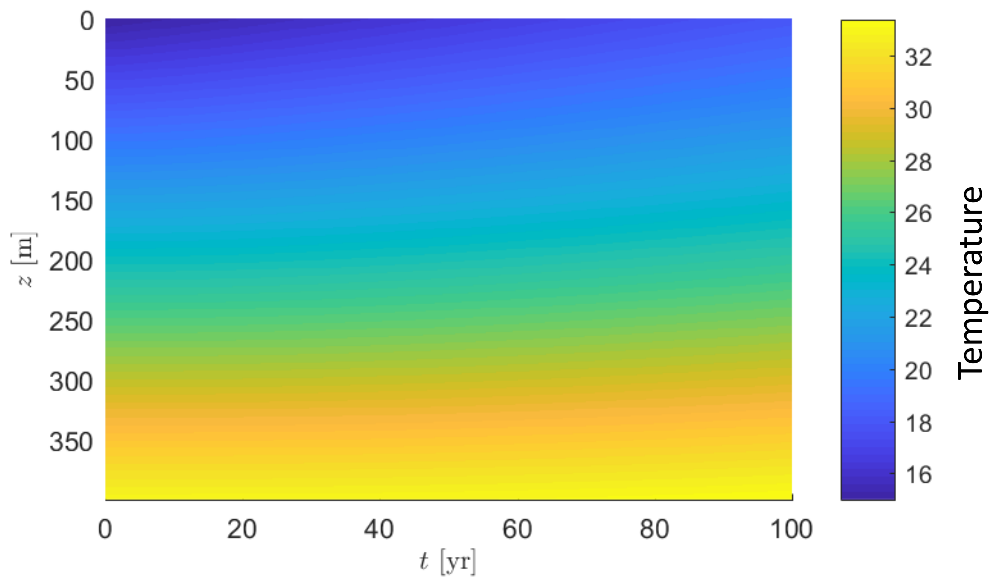
$F_{max} = 3°\text{C}$



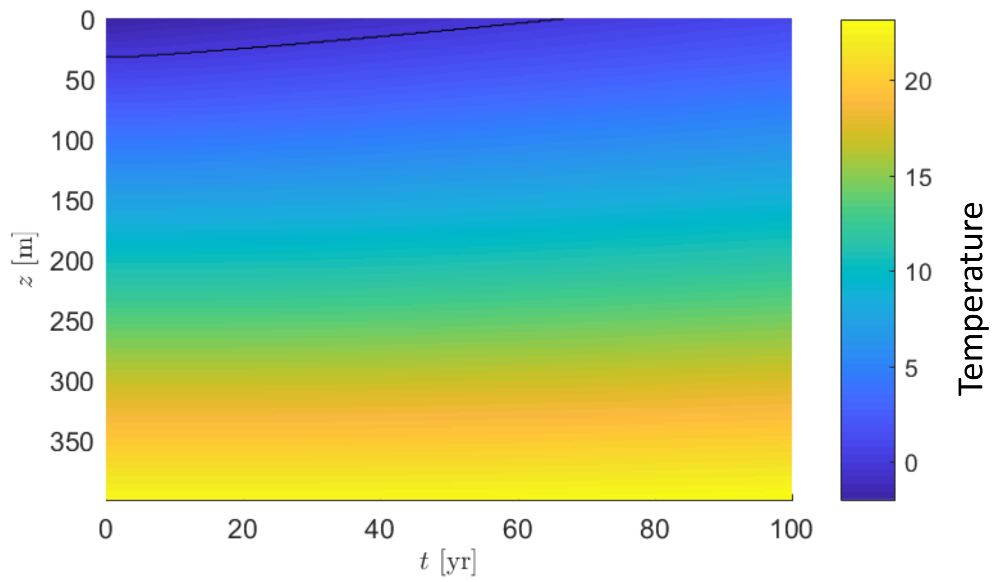
$(k = 700,\ M = 60,\ L = 1{,}000)$

Heat conduction as a model for permafrost

With no sinusoidal variation



$(T(y, 0) = 15)$



$(T(y, 0) = -2)$

Heat conduction as a model for permafrost

With added forcing, we can simulate the permafrost melting:

$$\frac{\partial T_y}{\partial t} = k\frac{\partial^2 T_y}{\partial z^2}, \qquad t \geq 0, \qquad 0 \leq z \leq L$$

$$T_y(0,t) = T(y,t) + F(t) \approx (-5 - 20\cos(2\pi t)) + \frac{F_{max}t}{t_{max}}$$

$$T_y(L,t) = M$$

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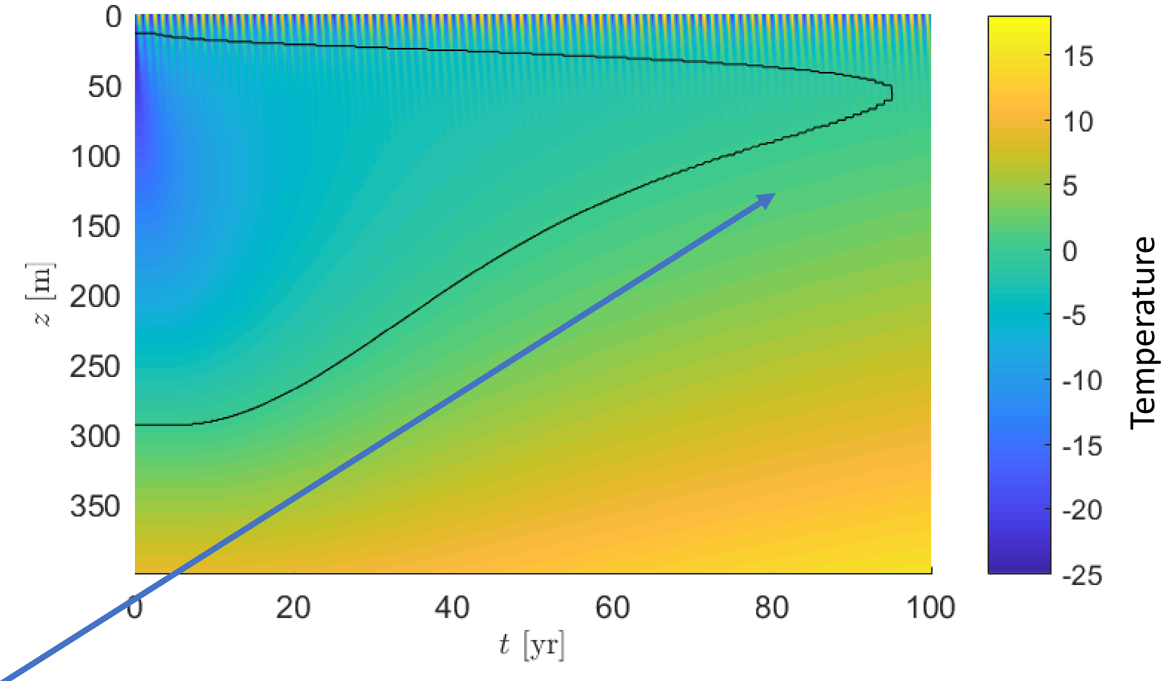


Signal for permafrost craters?

$(k = 700,\ M = 60,\ L = 1{,}000)$

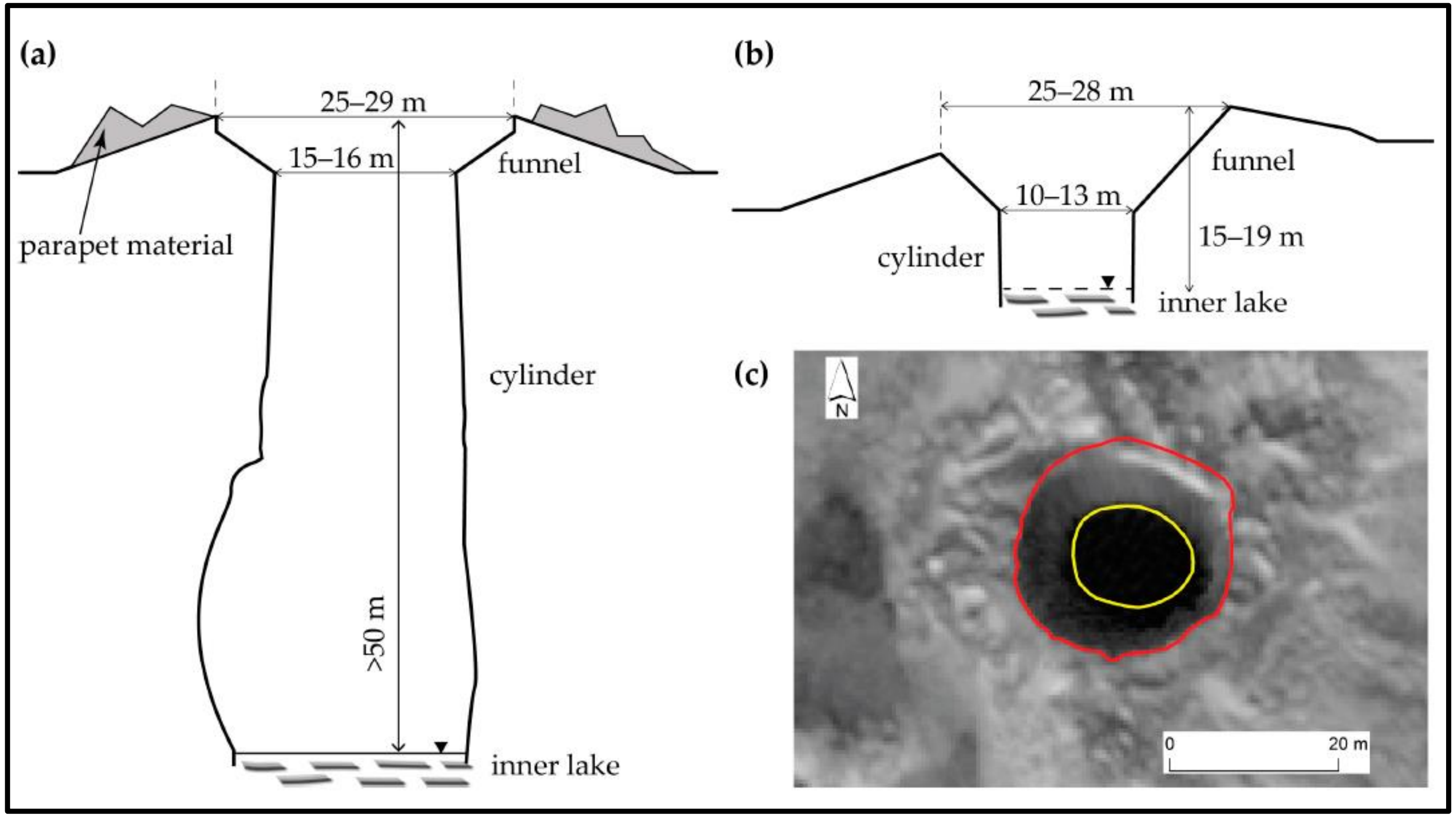
Heat conduction as a model for permafrost

Since 2014, multiple observations of permafrost craters in Siberia (Yamal Peninsula +)

(Leibman et al 14)

Heat conduction as a model for permafrost

Several studies have followed:



(Kizyakov 17)

Heat conduction as a model for permafrost

Several studies have followed:

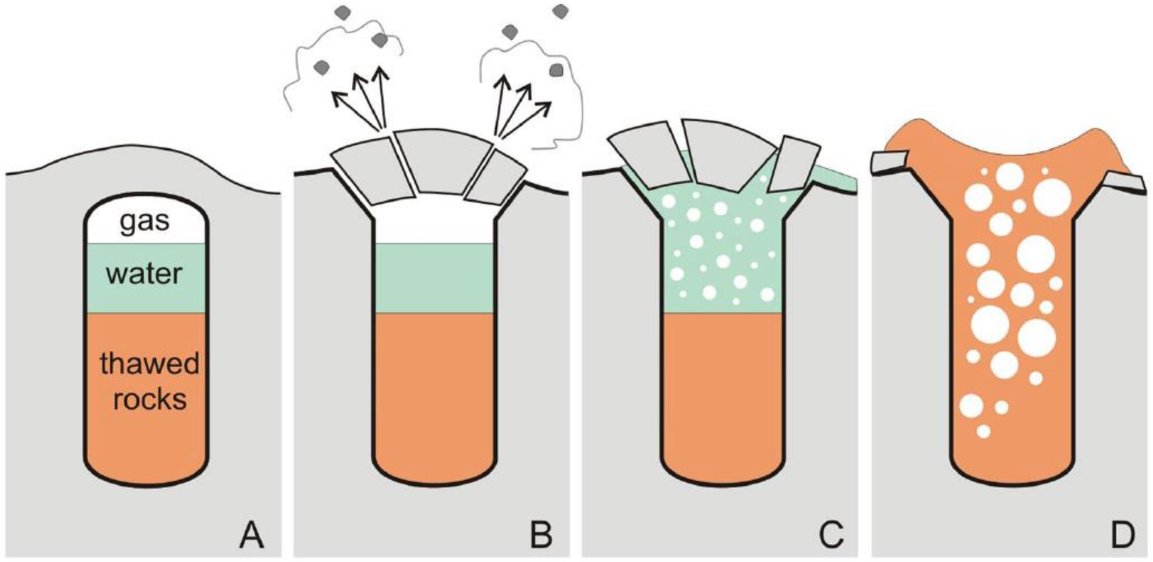


Figure 3. Pre-explosion (**A**), pneumatic (**B**), hydraulic (**C**), and phreatic (**D**) stages of a cryogenic eruption.

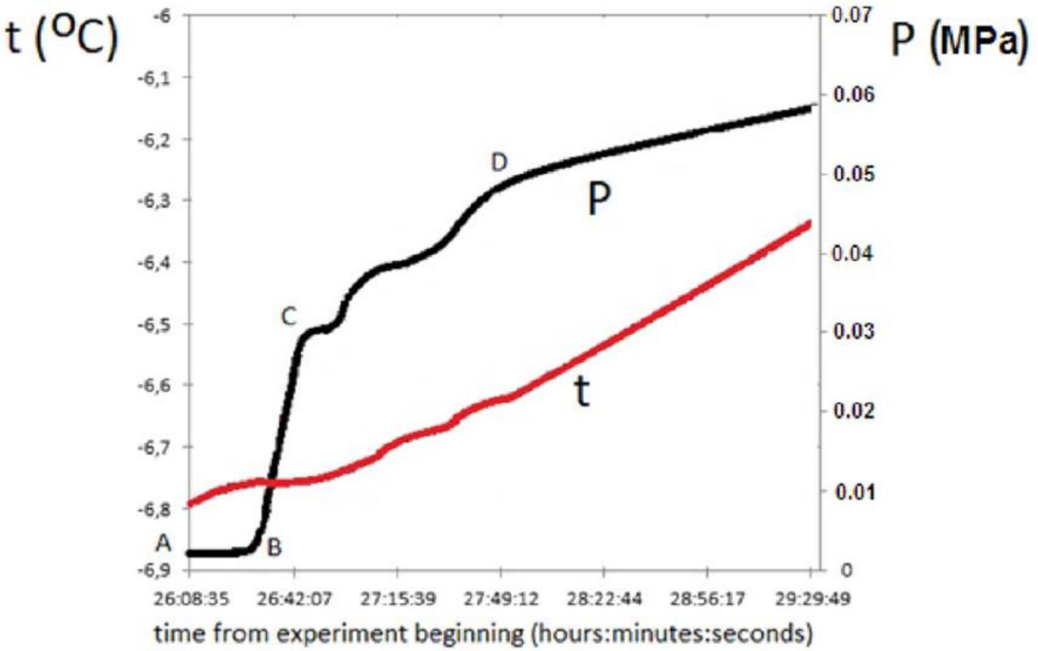


Fig. 4. Experimental curves of pressure (P) and temperature (t) changes inside the cell when slow heating of hydrate-containing sample.

(Buldoviz 18)

(Yakushev 18)

Heat conduction as a model for permafrost

Could the heat equation be enough to reproduce this effect?

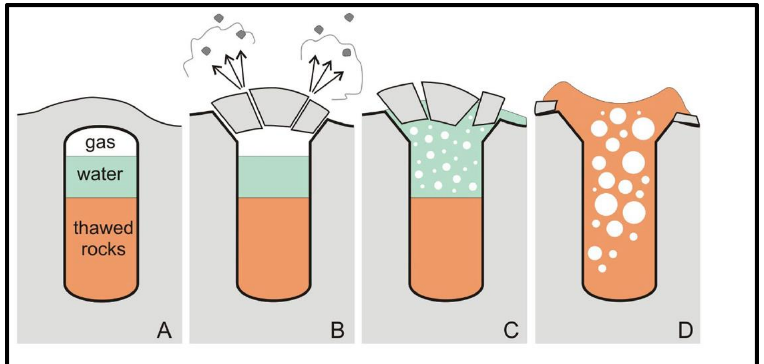
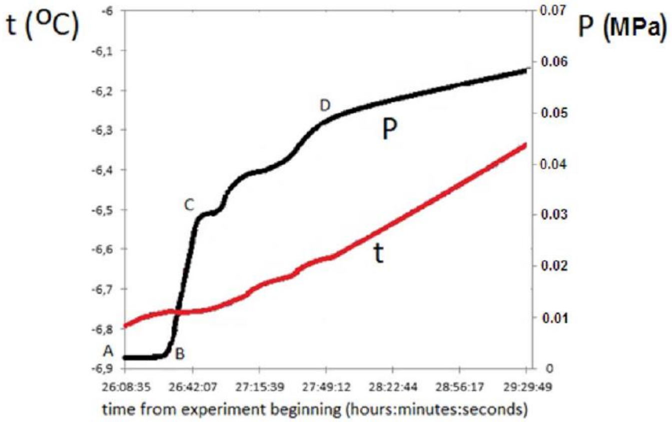
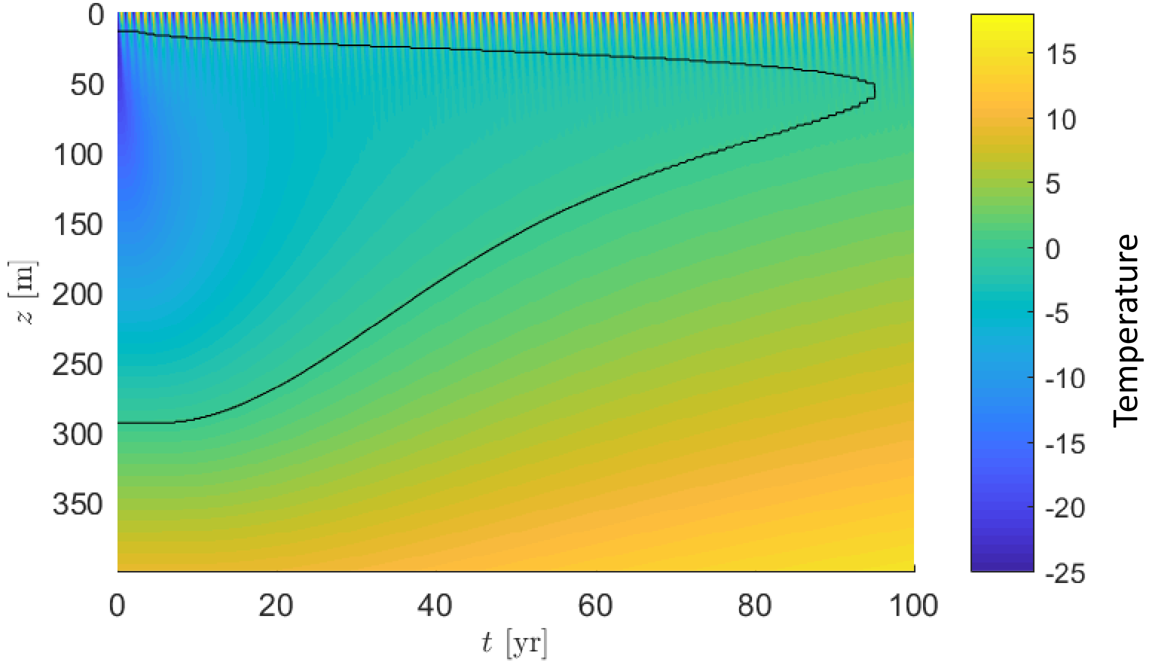


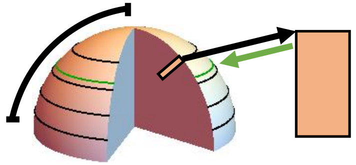
Figure 3. Pre-explosion (**A**), pneumatic (**B**), hydraulic (**C**), and phreatic (**D**) stages of a cryogenic eruption.





(Buldoviz 18, Yakushev 18)

In progress: coupling Budyko's model to heat equation

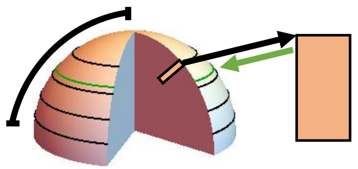


Together, the system is given by:

$$R\frac{\partial T}{\partial t} = \big(1-\alpha(y,\eta)\big)Qs(y) \;-\; (A+BT) \;+\; C(\bar{T}-T)$$

$$\frac{\partial T_y}{\partial t} = k\frac{\partial^2 T_y}{\partial z^2}$$

In progress: coupling Budyko's model to heat equation



Together, the system is given by:

$$R\,\frac{\partial T}{\partial t} = \big(1-\alpha(y,\eta)\big)Qs(y) \;-\; (A+BT) \;+\; C(\bar{T}-T)$$

$$\frac{\partial T_y}{\partial t} = k\,\frac{\partial^2 T_y}{\partial z^2}$$

At each latitude,

- Melting permafrost releases greenhouse gases (CO_2/methane)
- Once all gases from a latitude 'reservoir' are released, stop releasing

Thank you!

In collaboration with:

Richard McGehee

Aileen Zebrowski

John Nguyen

Heat conduction as a model for permafrost

Thermal diffusivity:

$$k = \frac{\alpha}{\rho c}$$

← Thermal conductivity (α)

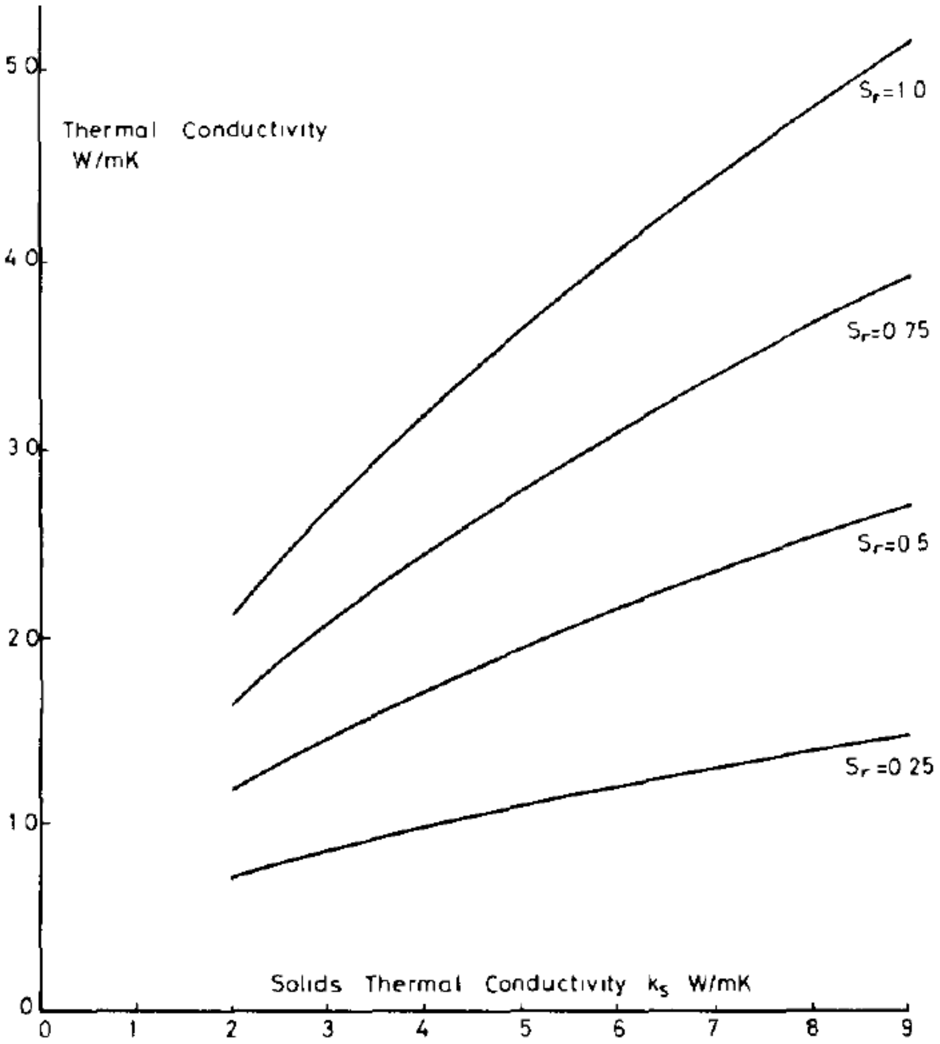
← Volumetric heat capacity (ρc)

$k \in (75, 828)$ m²/yr

Crust heat source:

$M \approx 30$°C (at $\mathrm{L} \approx 1{,}000$ m)

(Farouki 81, Lachenbruch 86)



S_r : saturation (% ice)